

AD-P001608

## COMPONENT PART NOTICE

THIS PAPER IS A COMPONENT PART OF THE FOLLOWING COMPILATION REPORT:

(TITLE): Behaviour of Short Cracks in Airframe Components, Conference Proceedings of the Meeting of the AGARD Structures and Materials Panel (55th) Held at Toronto, Canada on 19-24 September 1982.

(SOURCE): Advisory Group for Aerospace Research and Development, Neuilly-sur-Seine (France).

TO ORDER THE COMPLETE COMPILATION REPORT USE AD-A131 159.

THE COMPONENT PART IS PROVIDED HERE TO ALLOW USERS ACCESS TO INDIVIDUALLY AUTHORED SECTIONS OF PROCEEDINGS, ANNALS, SYMPOSIA, ETC. HOWEVER, THE COMPONENT SHOULD BE CONSIDERED WITHIN THE CONTEXT OF THE OVERALL COMPILATION REPORT AND NOT AS A STAND-ALONE TECHNICAL REPORT.

THE FOLLOWING COMPONENT PART NUMBERS COMPRISE THE COMPILATION REPORT:

AD# P001 602. TITLE: Mechanics and Physics of the Growth of Small Cracks.  
P001 603 ✓ Fatigue Damage Mechanisms and Short Crack Growth.  
P001 604 ✓ An Assessment of the Importance of Small Crack Growth to Aircraft Design.  
P001 605 ✓ A Nonlinear Fracture Mechanics Approach to the Growth of Small Cracks.  
P001 606 ✓ Damage Tolerance Evaluation of Structures with Small Cracks.  
P001 607 ✓ Fracture Mechanics Analysis of Short Cracks at Loaded Holes.  
P001 608 Probabilistic Fracture Mechanics Analysis Methods for Structural Durability.  
P001 609 ✓ The Effects of Compressive Overloads on the Threshold Stress Intensity for Short Cracks.  
P001 610 ✓ Crack Propagation at Short Crack Lengths under Variable Amplitude Loading (2nd Report).  
P001 611 ✓ Spectrum Effects on the Growth of Short Cracks.  
P001 612 ✓ A Study of Small Crack Growth under Transport Spectrum Loading.  
P001 613 ✓ Small Cracks in Large Forgings.  
P001 614 ✓ 'Short Crack' Fatigue Design Considerations: Modelling, Characterisation Interpretation, Detection: Prediction of Behavior.

### DISTRIBUTION STATEMENT A

Approved for public release;  
Distribution Unlimited

# PROBABILISTIC FRACTURE MECHANICS ANALYSIS METHODS FOR STRUCTURAL DURABILITY

JAMES L. RUDD  
Aerospace Engineer

Air Force Wright Aeronautical Laboratories (FIBEC)  
Wright-Patterson AFB, Ohio 45433, USA

JANN N. YANG  
Professor

George Washington University  
School of Engineering and Applied Science  
Washington, D.C. 20052, USA

SHERRELL D. MANNING  
Engineering Specialist Sr.

BILL G.W. YEE  
Chief of Materials Research Laboratory

General Dynamics Corporation  
Fort Worth Division  
P.O. Box 748  
Fort Worth, Texas 76101, USA

The United States Air Force requires that Air Force aircraft be designed to be durable. This requirement necessitates an analytical demonstration that excessive cracking within the airframe will not occur during the aircraft's design service life. In order to predict the time at which excessive cracking occurs, an analysis is needed which is capable of predicting the distribution of crack sizes within the airframe at any point in time. Such an analysis was recently developed and is presented in this paper. The durability analysis is based on a fracture mechanics philosophy, combining a probabilistic format with a deterministic crack growth rate relationship. Essential elements of the methodology are presented, with emphasis on the statistical representation of the initial fatigue quality of the structure. The accuracy of the durability analysis is demonstrated by correlating analytical predictions with experimental results of a fighter full-scale test article as well as complex-splice specimens subjected to a bomber load spectrum.

## I INTRODUCTION

The United States Air Force structural integrity requirements for metallic airframes are specified in Reference 1. Of particular importance are the damage tolerance (Refs. 2 and 3) and durability (Refs. 3 and 4) requirements. The damage tolerance requirements ensure aircraft safety while the durability requirement is to minimize structural maintenance costs and functional impairment problems. Both sets of requirements are necessary to ensure the operational readiness of Air Force aircraft.

This paper addresses the durability aspects of structural integrity. The durability damage mode considered is fatigue cracking in fastener holes. This was found to be a very prevalent form of degradation in aircraft structures (Ref. 5). Aircraft structural durability involves many fastener holes in various components which are susceptible to cracking in service. The associated structural maintenance costs are proportional to the number of fastener holes requiring repair. Therefore, to assess the durability of the structure or the extent of damage as a function of time, the entire population of fastener holes must be considered. Thus, a statistical approach is best suited for quantifying the extent of damage as a function of time.

Various aspects of structural durability have been considered in the literature (Refs. 6-27). Structural durability can be defined in many different ways. The most appropriate definition is dependent on the particular aircraft considered. The crack sizes of interest are a function of the definition used. This paper considers relatively small subcritical crack sizes which often affect durability. For example, 0.76 mm - 1.27 mm (0.03 inch - 0.05 inch) radial cracks in fastener holes can affect functional impairment, structural maintenance requirements, and life-cycle costs. Such cracks do not pose an immediate safety problem. However, if the fastener holes containing such cracks are not repaired, economical repairs cannot be made when these cracks exceed a limiting crack size. For example, a 0.76 mm - 1.27 mm radial crack in a fastener hole can be cleaned up by reaming the hole to the next nominal hole size. When the crack sizes exceed this economical repair limit, excessive maintenance and repair costs can occur. Hence, an analysis is needed for predicting the extent of damage present at any particular point in time. Such an analysis was recently developed (Refs. 11-16 and 18-27) and evaluated for load transfer coupon specimens (Refs. 11, 12, 15, 21, 22, and 27).

AD-P001608

10-2 This paper presents the recently developed durability analysis methodology. The methodology is based on a probabilistic fracture mechanics approach. The initial fatigue quality of a structure is represented in a statistical manner by a distribution of equivalent initial flaw sizes. These equivalent initial flaw sizes are grown forward in time using a deterministic crack growth rate relationship. An evaluation is made of the accuracy of the analysis by correlating analytical predictions with test data for a fighter full-scale test article (Refs. 21 and 22) and complex splice specimens subjected to a bomber load spectrum (Ref. 27).

## II DURABILITY DESIGN REQUIREMENTS

Air Force durability design requirements for metallic airframes are presented in References 1, 3 and 4. According to these requirements, the airframe must be designed to have an economic life greater than the design service life. Furthermore, the economic life must be demonstrated by analysis and test. The economic life analysis must account for the effects of initial quality variations, material property variations and the design loads/environments.

The economic life of a structure is currently defined in only qualitative terms: "... the occurrence of widespread damage which is uneconomical to repair and, if not repaired, could cause functional problems affecting operational readiness" (Ref. 1). There is no universal quantitative definition of "widespread damage" or acceptable structural maintenance cost limits. Such limits are currently determined by the contractor and the Air Force for the particular aircraft of interest. Durability compliance standards are defined based on the results of the full-scale durability test article.

## III DURABILITY ANALYSIS CRITERIA

### A. Durability Critical Parts Criteria

Criteria must be developed for determining which parts of an aircraft are durability critical (i.e., which parts must be designed to meet the durability design requirements). The durability critical parts criteria vary from aircraft to aircraft. They are especially dependent on the definition of economic life for the particular aircraft involved. A more detailed discussion of the durability critical parts criteria is presented elsewhere (Ref. 12). A typical flow diagram for selecting which parts are durability critical is presented in Fig. 1. In Fig. 1, durability refers to the ability of an airframe to resist crack growth whereas damage tolerance refers to the ability of an airframe to resist failure due to the presence of such cracks.

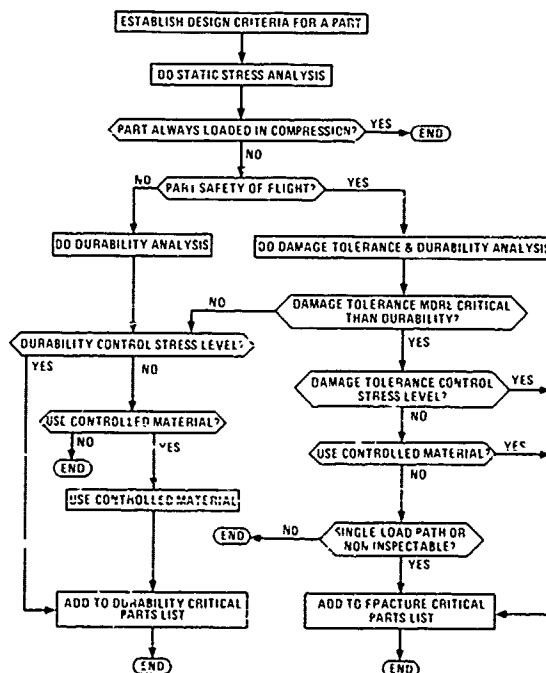


Figure 1 - Flow diagram for selecting durability critical parts

## B. Economic Life Criteria

Criteria must be developed for determining the economic life of the particular aircraft of interest. Similar to the durability critical parts criteria, economic life criteria vary from aircraft to aircraft. They may be based on fastener hole repair (e.g., reaming the damaged fastener hole to the next nominal hole size), functional impairment (e.g., fuel leakage), residual strength, etc. Two promising analytical formats for quantifying the economic life of an airframe are (1) the probability of crack exceedance, and (2) cost ratio: repair cost/replacement cost. Both formats require a durability analysis methodology capable of quantifying the extent of aircraft structural damage as a function of service time. For example, assume the economic life criteria are based on the number of fastener holes which cannot be economically repaired (i.e., number of fastener holes with crack sizes equal to or greater than specified size  $X_1$ ). Then an analytical format for quantifying economic life is presented in Fig. 2. In Fig. 2,  $P$  is the exceedance probability. More detailed discussions of economic life criteria are presented elsewhere (Refs. 11, 12, 15, and 18).

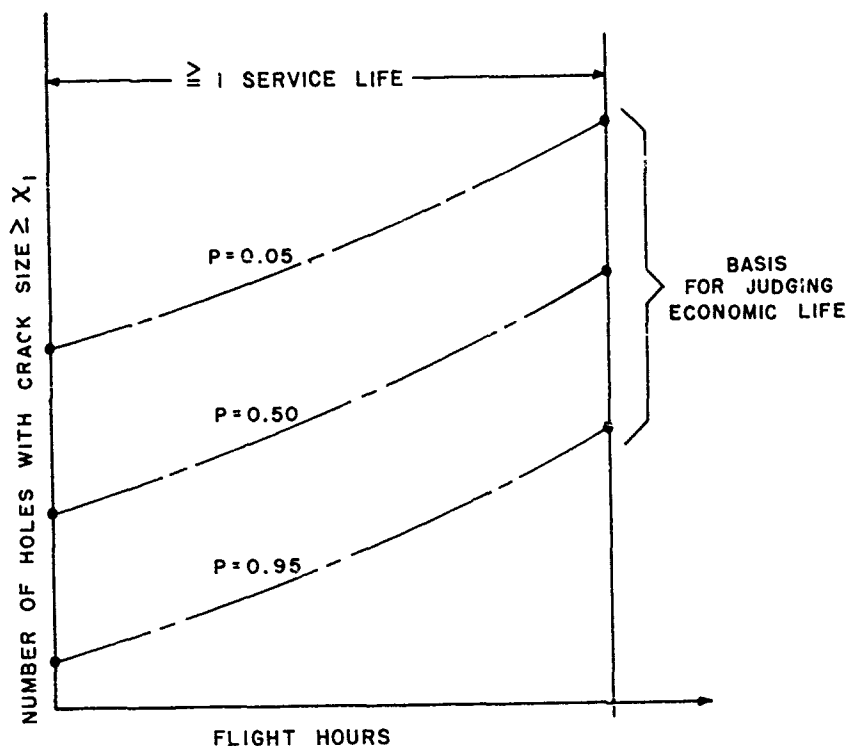


Figure 2 - Analytical format for economic life

## IV DURABILITY ANALYSIS METHODOLOGY

### A. General Description

The basic objective of the durability analysis methodology is to quantify the extent of damage as a function of service time for a given aircraft. The extent of damage is measured by the number of structural details (e.g., fastener holes, cut-outs, fillets, lugs, etc.) expected to have a crack size greater than a specified size at a given service time. Hence, the extent of damage is represented by the probability of crack exceedance. The durability analysis results provide a quantitative description of the extent of damage and a basis for analytically assuring that the economic life of the structure will exceed the design service life.

The durability analysis includes two essential steps: (1) the quantification of the initial fatigue quality of the structural details considered, and (2) the prediction of the probability of crack exceedance based on the initial fatigue quality and the applicable design conditions (e.g., load spectrum, stress levels, percent load transfer, etc.).

## B. Initial Fatigue Quality

### 1. Initial Fatigue Quality Description

Initial fatigue quality (IFQ) defines the initial manufactured state of a structural detail or details with respect to initial flaws in a part, component or airframe prior to service. The IFQ for a group of replicate details is represented by an equivalent initial flaw size (EIFS) distribution. An equivalent initial flaw is a hypothetical crack assumed to exist in a detail prior to service. An equivalent initial flaw size is the initial size of a hypothetical crack which would result in an actual crack size at an actual point in time. An arbitrary crack size,  $a_0$ , is selected which can be readily detected or which can be reliably observed fractographically following testing. The time required for an initial defect, of whatever type, to become a fatigue crack of size  $a_0$  is defined as the time-to-crack-initiation (TTCI). Test results of TTCI and crack growth rates using coupon specimens are employed to define the EIFS distribution. A conceptual description of the initial fatigue quality model is shown in Fig. 3 (Refs. 11, 12, 15, 21). Once the EIFS distribution has been defined for a group of details, a deterministic crack growth analysis is used to grow the entire EIFS population to any service time  $t$ ; thus determining the time-varying crack size distribution in service (Fig. 4).

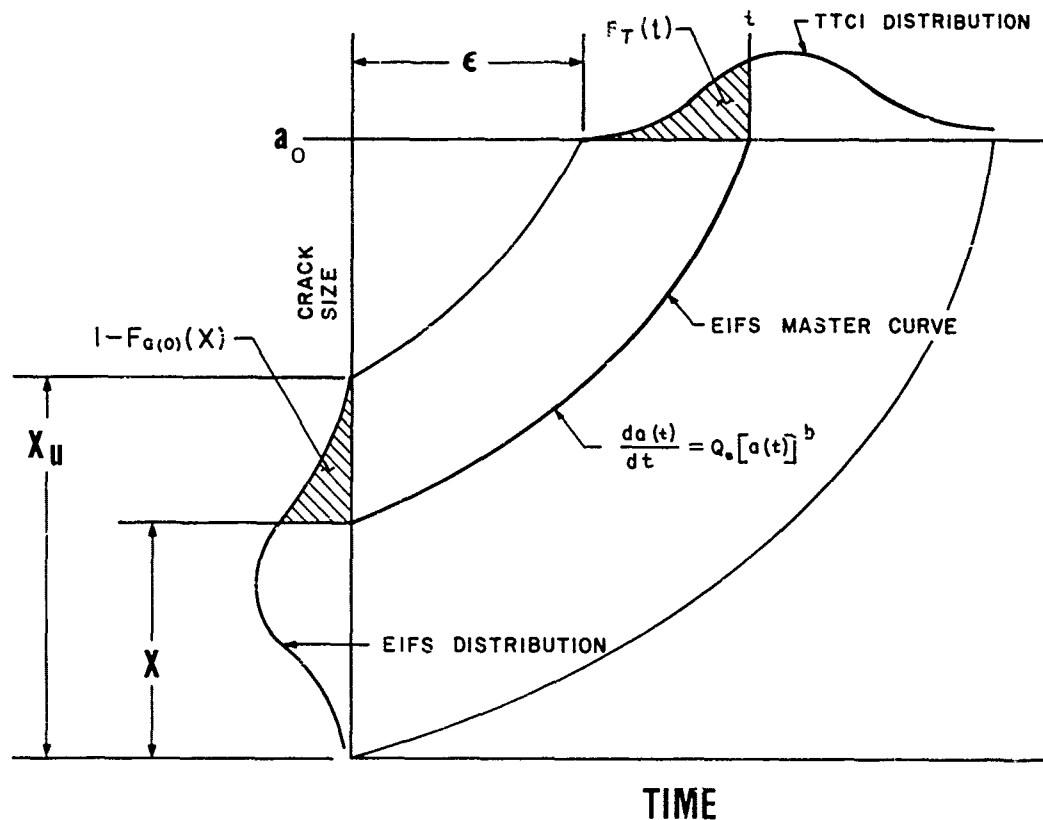


Figure 3 - Initial fatigue quality model

### 2. Initial Fatigue Quality Model

A prototype initial fatigue quality (IFQ) model has been previously described for quantifying the EIFS distribution for structural details, such as fastener holes (Refs. 11, 15, and 16). Such a model is conceptually described in Fig. 3. IFQ model refinements and EIFS distributions for two model variations are summarized in the following.

The TTCI distribution for coupon specimens is represented by the three-parameter Weibull distribution,  $F_T(t)$  (Refs. 11, 15 and 16).

$$F_T(t) = P[T \leq t] = 1 - \exp \left[ - \left( \frac{t - \epsilon}{\beta_0} \right)^{\alpha_0} \right], \quad t > \epsilon \quad (1)$$

where  $T$  = TTCI,  $\alpha_0$  = shape parameter,  $\beta_0$  = scale parameter and  $\epsilon$  = lower bound TTCI. The three Weibull parameters (i.e.,  $\alpha_0$ ,  $\beta_0$ ,  $\epsilon$ ) are determined from fractography for coupon specimens or other suitable test results.

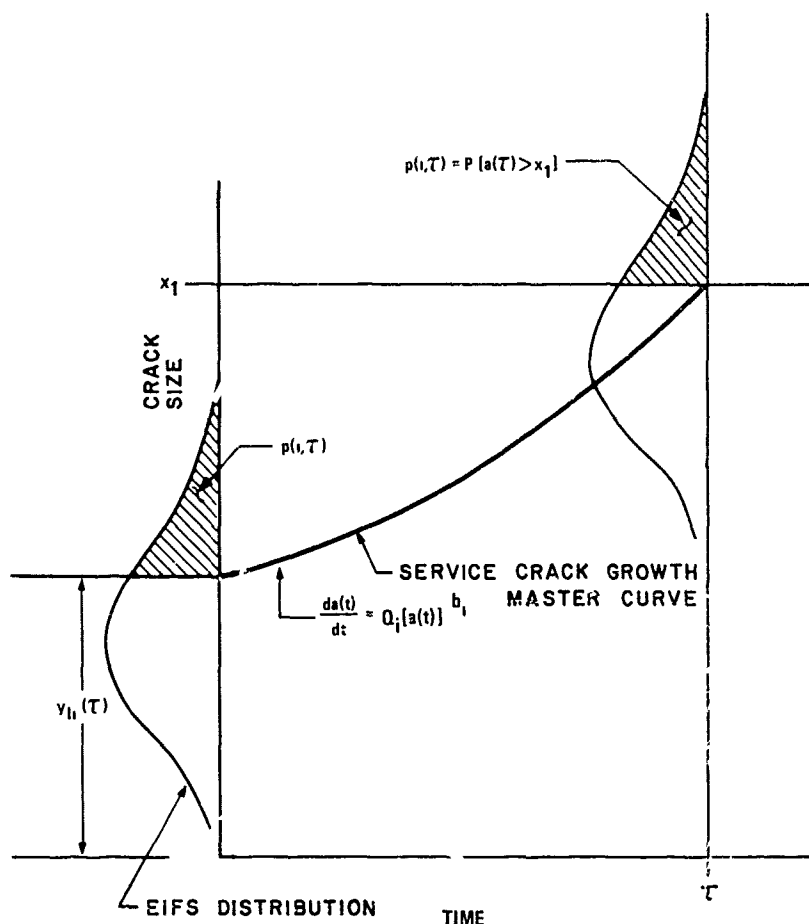


Figure 4 - Growth of EIFS distribution as function of time

The crack growth rate in the small crack region is assumed to be of the following form

$$\frac{da(t)}{dt} = Q_0 [a(t)]^b \quad (2)$$

where  $Q_0$  and  $b$  are parameters depending on loading spectra, structural and material properties, etc.;  $a(t)$  is the crack size at time  $t$ . Other functional forms for the crack growth rate could also be used. Equation 2 is used because of its simplicity and general applicability for matching the crack growth data. A crack growth rate equation, such as Eq. 2, is used to obtain the EIFS distribution through a transformation of the TTCI distribution; hence both EIFS and TTCI are statistically compatible. EIFS distributions are obtained using Eq. 2 for  $b \neq 1$  (case I) and  $b = 1$  (case II). The resulting EIFS distribution for both cases are described below.

a. Case I ( $b \neq 1$ )

Integrating Eq. 2 from  $t = 0$  to  $t = T$ , the relationship between the initial crack size,  $a(0)$ , and the reference crack size,  $a_0 = a(T)$  for TTCI (flight hours), is obtained,

$$\text{EIFS} = a(0) = [a_0^{-c} + cQ_0T]^{-1/c} \quad (3)$$

where  $c = b - 1$ .

The EIFS cumulative distribution,  $F_{a(0)}(x)$ , is obtained by combining Eqs. 1 and 3 as follows:

$$F_{a(0)}(x) = \exp \left\{ - \left[ \frac{x^{-c} - a_0^{-c} - cQ_0T}{cQ_0T} \right]^{\alpha_0} \right\} ; \quad x \leq x_u \quad (4)$$

$$= 1; \quad x \geq x_u$$

10-6 where

$$x_u = [a_0^{-c} + cQ_0\epsilon]^{-1/c} \quad (5)$$

is the upper bound of the EIFS distribution. The lower bound of the TTCI distribution,  $\epsilon$ , for a given reference size,  $a_0$ , is obtained from Eq. 5 as follows:

$$\epsilon = \frac{1}{cQ_0} [x_u^{-c} - a_0^{-c}]; \quad a_0 \geq x_u \quad (6)$$

Equation 4 for  $F_{a(0)}(x)$  can be simplified by substituting the expression for  $cQ_0\epsilon$  from Eq. 5 into Eq. 4 as follows:

$$F_{a(0)}(x) = \exp \left\{ - \left[ \frac{x_u^{-c} - x^{-c}}{cQ_0\beta_0} \right]^{\alpha_0} \right\}; \quad x \leq x_u \quad (7)$$

$$= 1; \quad x \geq x_u$$

b. Case II ( $b = 1$ )

Integrating Eq. 2 from  $t = 0$  to  $t = T$  and considering  $b = 1$ , one obtains the relationship between the initial crack size,  $a(0)$ , and the reference crack size,  $a_0$ , as follows

$$\text{EIFS} = a(0) = a_0 \exp(-Q_0 T) \quad (8)$$

The EIFS cumulative distribution,  $F_{a(0)}(x)$ , is obtained by combining Eqs. 1 and 3 as follows:

$$F_{a(0)}(x) = \exp \left\{ - \left[ \frac{\ln(x_u/x)}{Q_0\beta_0} \right]^{\alpha_0} \right\}; \quad 0 < x \leq x_u \quad (9)$$

$$= 1; \quad x \geq x_u$$

where  $x_u$  is the upper bound of the EIFS distribution:

$$x_u = a_0 \exp(-Q_0\epsilon) \quad (10)$$

The lower bound of TTCI distribution,  $\epsilon$ , for a given reference crack size,  $a_0$ , is obtained from Eq. 10 as follows:

$$\epsilon = \frac{1}{Q_0} \ln(a_0/x_u); \quad a_0 \geq x_u \quad (11)$$

When each test specimen consists of 2 fastener holes equally stressed and fractography results are taken only for the fastener hole in each specimen which has the largest crack size, the exponential exponents of Eqs. 1, 4, 7 and 9 should be multiplied by a factor of 1/2 (Ref. 22).

## 2. Generic EIFS and Discussion of IFQ Model

Intuitively, the EIFS cumulative distribution,  $F_{a(0)}(x)$ , should be only a function of the material and the manufacturing/fabrication processes. As such  $F_{a(0)}(x)$  (Eqs. 4, 7 and 9) should be independent of loading spectra, stress level, percent shear load transfer through the fasteners, etc. If the same EIFS cumulative distribution is valid for replicate fastener holes under different design conditions (e.g., loading spectra, stress level, & load transfer, etc.), then the resulting  $F_{a(0)}(x)$  is said to be "generic". If  $F_{a(0)}(x)$  is generic, then the crack growth damage accumulation can be calculated analytically or numerically for different design conditions using the EIFS distribution (Eqs. 4, 7 or 9).

The IFQ model parameters,  $\beta_0$ ,  $\beta$ ,  $Q_0$ , and  $b$  depend on the fractographic results. Therefore, these parameters depend on the conditions used to generate the crack initiation and crack growth data (e.g., material, loading spectra, stress level, etc.). A basic premise of the durability analysis method proposed is that  $F_{a(0)}(x)$  can be determined from a given fractography data set or sets economically and the resulting EIFS distribution can be used for crack exceedance predictions for different loading spectra,

stress levels, % load transfer, etc. Encouraging results have been obtained to date which suggest that such a basic premise appears to be promising. However, further study is required to evaluate the IFQ distributions using available fractographic data generated in Ref. 23 and to assess the accuracy of the crack exceedance predictions,  $p(i, \tau)$ , under different design conditions.

For simplicity, suppose two sets of replicate specimens are tested using the same loading spectrum but different stress levels. Using the fractographic results for each data set, the respective TTCIs for a given  $a_0$  can be determined for each data set as illustrated in Fig. 5. For the EIFS cumulative distribution,  $F_{a(0)}(x)$ , to be "generic", the TTCI distributions,  $F_T(t)$ , for data sets 1 and 2 should transform into the same EIFS distribution. From Eq. 9, the necessary conditions for a generic EIFS cumulative distribution are for  $\alpha_0$  and  $Q_0\beta_0$  to be constants.

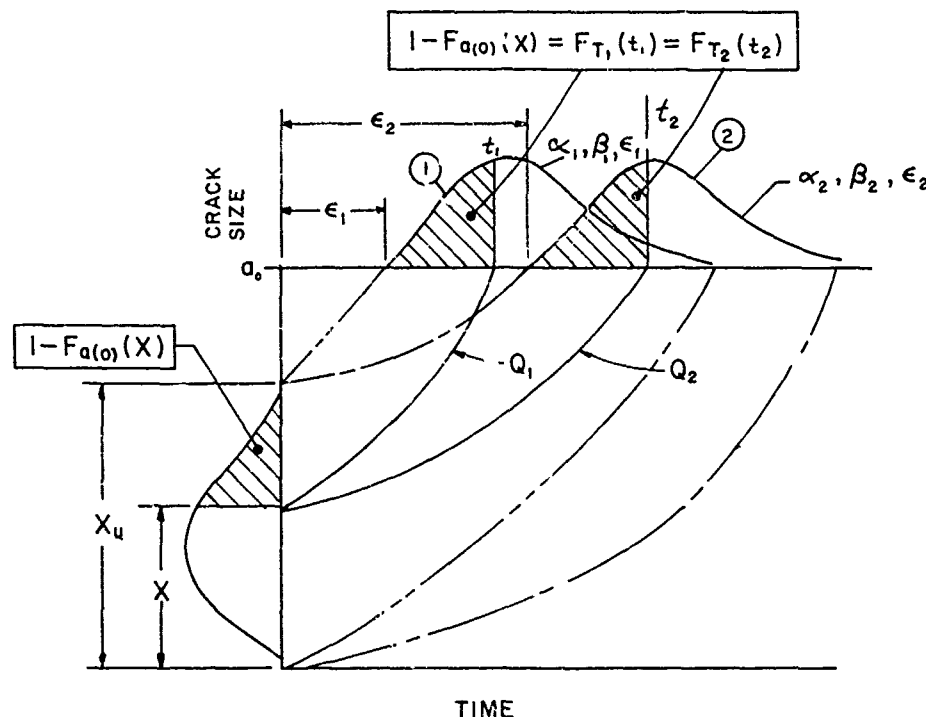


Figure 5 - Generic EIFS condition

Investigations have shown that  $\alpha_0$  is a material constant for a given type of fastener hole and the product  $Q_0\beta_0$  appears to be the common denominator for linking different fractographic data sets together on a common baseline. Although results to date are very encouraging, further research is required to evaluate and compare  $Q_0\beta_0$  values for different fractographic data sets.

The EIFS distribution for case I ( $b \neq 1$ ) and case II ( $b = 1$ ) are given by Eqs. 7 and 9, respectively. In both cases, the EIFS cumulative distribution,  $F_{a(0)}(x)$ , is independent of the reference crack size,  $a_0$ , used to define the TTCI values. This is an important attribute of the IFQ distribution. The upper bound EIFS value in the IFQ distribution,  $X_u$ , is either selected by the user or computed from Eq. 5 or 10. The other parameters (i.e.,  $\alpha_0$ ,  $\beta_0$ ,  $\epsilon$ ,  $Q_0$  and  $b$ ) are determined from fractographic results for coupon specimens or other suitable test results.

In previous work, Case I ( $b \neq 1$ ) has been considered (Refs. 11, 12, 16, 21, and 22). Fractographic results for protruding head (Ref. 33) and countersunk head (Ref. 23) fasteners resulted in  $b$  values  $< 1$ . When  $b$  is  $< 1.0$ , it is possible to obtain EIFS values  $\leq 0$  using the original IFQ model. Mathematically, however, the original IFQ model can handle both positive and negative EIFS values, since the IFQ model is simply a "mathematical tool" for predicting the probability of crack exceedance. An explanation of the negative EIFS issue in terms of the original IFQ model and the crack initiation process is given in Ref. 22.



10-8

There are certain advantages and disadvantages associated with the use of Case I ( $b \neq 1$ ) and Case II ( $b = 1$ ) in Eq. 2. For example, the EIFS master curve for Case I with two parameters  $Q_0$  and  $b \neq 1$  generally fit the selected fractographic crack size range better than Case II with a single parameter ( $Q_0, b = 1$ ). However, the resulting  $Q_0$  and  $b$  values must be on a comparable baseline when different fractography data sets are considered. Since the resulting  $Q_0$  and  $b$  values affect the IFQ distribution, they must be consistently defined. In Eq. 2,  $Q_0$  and  $b$  are shown to be strongly correlated parameters (Ref. 19). Hence, for a given  $b$  there is likely to be a corresponding  $Q_0$  and vice versa. Thus, for consistent IFQ results, the same  $b$  value may be determined using pooled fractography results for different data sets.

When the one parameter form of Case II is used in Eq. 2 (i.e.,  $Q_0, b = 1$ ), the " $Q_0$ " value for one fractography data set is already comparable with the " $Q_0$ " value for another data set. Nevertheless, whichever form of Eq. 2 is used (i.e.,  $Q_0, b \neq 1$  or  $Q_0, b = 1$ ) the resulting  $F_a(0)(x)$  will be statistically compatible with the TTCI distribution. Consequently, as long as the resulting  $F_a(0)(x)$  is used in a consistent manner, the same crack exceedance prediction will be obtained.

### C. Durability Analysis Procedures

The durability analysis procedures, described and discussed in detail elsewhere (Refs. 15, 21 and 22), are summarized below for Case II ( $b = 1$ ). Similar expressions can be developed for Case I ( $b \neq 1$ ) using the same procedures.

(1) Divide the durability component into  $m$  stress regions where the maximum stress in each region may be reasonably assumed to be equal for every location or detail (e.g., fastener hole).

(2) Use the model shown in Fig. 3 and suitable fractography results to define the EIFS distribution expressed in Eq. 9. Determine  $\alpha_0$  and  $Q_0 \beta_0$  (Ref. 22). The  $x_u$  selected should be consistent with Eq. 10.

(3) Determine for each stress region, the corresponding EIFS value,  $y_{1i}(\tau)$ , which grows to crack size  $x_1$  at service time  $\tau$  as illustrated in Fig. 4 (Ref. 22). If applicable fractography data are available for different stress levels and fractography data pooling procedures are used, the crack growth rate expression in Eq. 12, where  $b_i = 1$ , can be integrated from  $a(0) = y_{1i}(\tau)$  to  $a(\tau) = x_1$  to obtain  $y_{1i}(\tau)$  in Eq. 13.

$$\frac{da(t)}{dt} = Q_i [a(t)]^{b_i} \quad (12)$$

$$y_{1i}(\tau) = x_1 \exp(-Q_i \tau) \quad (13)$$

in which

$$Q_i = \xi \sigma^\gamma \quad (14)$$

is assumed to be a power function of the maximum applied stress  $\sigma$ , and  $\xi$  and  $\gamma$  are constants to be determined from available fractography data.

If applicable fractographic results are not available for the desired design conditions (e.g., load spectra, % load transfer, stress level, etc.), an analytical crack growth program (e.g., Ref. 36) can be used to generate a "service crack growth master curve" to determine  $y_{1i}(\tau)$  for a given  $x_1$  (Refs. 11 and 15). When an analytical crack growth program is used,  $y_{1i}(\tau)$  must be determined in a manner which is consistent with that used to determine the EIFS distribution from the fractographic test data.

(4) Compute the probability of crack exceedance for each stress region, i.e.,  $p(i, \tau) = P[a(\tau) > x_1] = 1 - F_a(0)[y_{1i}(\tau)]$ , using Eq. 9.

$$p(i, \tau) = 1 - \exp \left\{ - \frac{1}{\ell} \left[ \frac{\ln(x_u / y_{1i}(\tau))}{Q_0 \beta_0} \right]^{\alpha_0} \right\}; \quad 0 < y_{1i} \leq x_u \quad (15)$$

$$p(i, \tau) = 0; \quad y_{1i}(\tau) \geq x_u$$

in which  $y_{1i}(\tau)$  is given by Eq. 13, and  $\ell$  is the scaling factor described previously based on the number of fastener holes per specimen used in the fractography data base.

(5) The average number of details  $\bar{N}(i, \tau)$ , and the standard deviation  $\sigma(i, \tau)$  in the  $i$ th stress region with a crack size greater than  $x_1$  at service time  $\tau$  are determined using the binomial distribution and are expressed as follows:

$$\bar{N}(i, \tau) = N_i p(i, \tau) \quad (16)$$

$$\sigma_N(i, \tau) = \{N_i p(i, \tau) [1 - p(i, \tau)]\}^{1/2} \quad (17)$$

in which  $N_i$  denotes the total number of details in the  $i$ th stress region. The average number of details with a crack size exceeding  $x_1$  at the service time  $\tau$  for  $m$  stress regions,  $\bar{L}(\tau)$ , and its standard deviation,  $\sigma_L(\tau)$ , can be computed using Eqs. 18 and 19.

$$\bar{L}(\tau) = \sum_{i=1}^m \bar{N}(i, \tau) \quad (18)$$

$$\sigma_L(\tau) = \left[ \sum_{i=1}^m \sigma_N^2(i, \tau) \right]^{1/2} \quad (19)$$

Equations 18 and 19 can be used to quantify the extent of damage for a single detail, a group of details, a part, a component, or an airframe. Upper and lower bounds for the prediction can be estimated using  $\bar{L}(\tau) \pm Z\sigma_L(\tau)$ , where  $Z$  is the number of standard deviations,  $\sigma_L(\tau)$ , from the mean,  $\bar{L}(\tau)$ . Equations 16 through 19 are valid if cracks in each detail are relatively small and the growth of the largest crack in each detail is not affected by cracks in neighboring details. Hence, the crack growth accumulation for each detail is statistically independent (Refs. 11, 12, 15 and 21).

## V. DURABILITY ANALYSIS DEMONSTRATIONS

A durability analysis of the lower wing skins of a fighter is presented. Durability analysis results for complex-splice specimens subjected to a bomber load spectrum are also presented. Both analyses are correlated with test data.

### A. Fighter Lower Wing Skins

A durability analysis of the lower wing skins of a fighter durability test article is presented to illustrate the methodology described. Analytical predictions of the extent of damage in each wing skin are presented in various formats, and results are compared with observations from the tear-down inspection of the fighter durability test article.

The fighter durability test article was tested to 16,000 flight hours (equivalent to 2 service lives) using a 500-hour block spectrum. Each wing received the same loading. Following the test, all fastener holes in the lower wing skins were inspected using eddy current techniques. Fastener holes with crack indications were confirmed by fractographic evaluation. The right hand and left hand lower wing skins were found to have twenty six and seven fastener holes, respectively, with a crack size  $\geq 0.76$  mm (0.03 inch) at  $\tau = 16,000$  flight hours.

A preliminary durability analysis for the fighter lower wing skins was presented in Ref. 21. The preliminary analysis reflected: (1) fastener hole IFQ based on fractographic results for protruding head fasteners, (2) crack growth rates for the IFQ model based on Eq. 2 ( $b \neq 1$ ), (3) three-parameter Weibull distribution used in the IFQ model, (4) model parameters based on a single data set (one stress level, 400-hour block spectrum, Ref. 33), (5) three stress regions considered for the lower wing skin, and (6) an analytical crack growth program (Ref. 36) and the 500-hour block spectrum were used to define the "service crack growth master curve" for each stress region.

Essential features of the present analysis are: (1) fractographic results for countersunk fasteners used to quantify IFQ (countersunk fasteners were used on the fighter durability test article), (2) crack growth rates for the IFQ model based on Eq. 2 ( $b = 1$ ), (3) three-parameter Weibull distribution used in the IFQ model, (4) model parameters based on three different data sets (three stress levels, 400-hour block spectrum) (5) lower wing skin divided into 10 stress regions, and (6) crack growth rate parameter  $Q_i$  defined for each stress region as a function of stress level  $\sigma_i$ , determined from available fractography data. There were no significant differences in the 400-hour and 500-hour spectra.

The fighter lower wing skin was divided into ten stress regions as shown in Fig. 6. Applicable stress levels and the corresponding number of fastener holes in each stress region are shown in Table 1. The stress levels for Zones I-IV were determined using strain gage data in combination with finite element analyses. The stress levels for Zones V, VII-IX were determined using a coarse grid finite element analysis and a theoretical stress distribution for a circular hole in an infinite plate under uniaxial tension. The stress levels for Zones VI and X were determined from a fine grid finite element analysis.

Fractographic results for three data sets (i.e., AFXLR4, AFXMR4, and AFXHR4) for maximum stress levels of 220.7 MPa (32 ksi), 234.5 MPa (34 ksi), and 262.1 MPa (38 ksi) were used to calibrate the IFQ model parameters. A 400-hour block load spectrum was used. The AFX series specimens were designed for 15% load transfer. The specimens were made of 7475-T7351 aluminum and contained two MS90353-08 ( $\frac{1}{4}$  dia.) blind, countersunk rivets as shown in Fig. 7. All specimens reflect typical aircraft production quality, tolerances and fastener fits. Nine specimens were tested per data set.

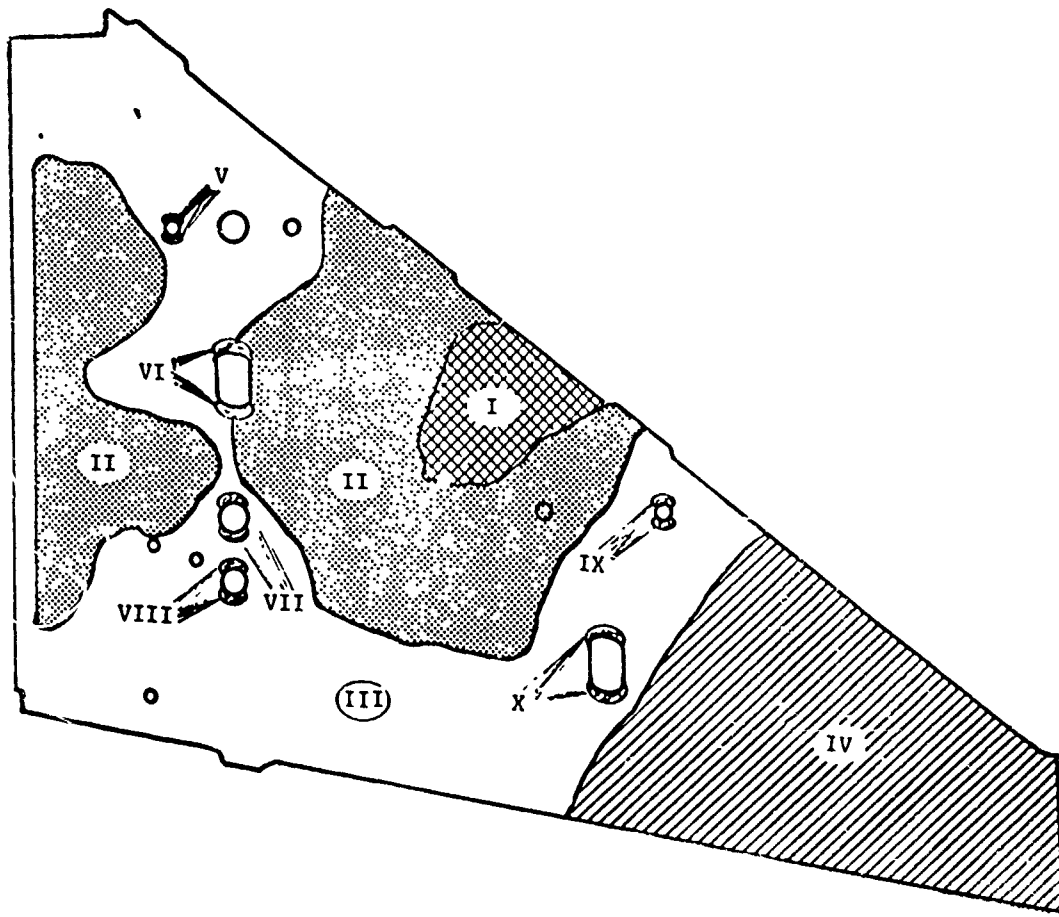


Figure 6 - Stress zones for fighter lower wing skin

Table 1 - Stress levels and number of fasteners holes for fighter lower wing skin

STRESS ZONE	LIMIT STRESS LEVEL, MPa (ksi)	NUMBER OF FASTENER HOLES
I	195.2 (28.3)	59
II	186.2 (27.0)	320
III	167.6 (24.3)	680
IV	115.2 (16.7)	469
V	195.9 (28.4)	8
VI	201.4 (29.2)	30
VII	223.5 (32.4)	8
VIII	180.7 (26.2)	8
IX	180.7 (26.2)	12
X	177.2 (25.7)	20
		1614

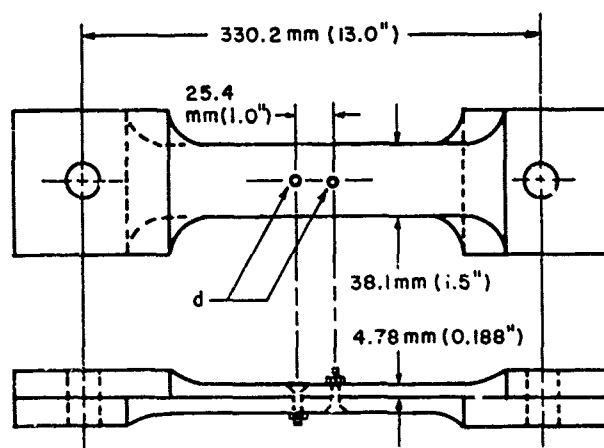


Figure 7 - IFQ specimen

The crack growth rate parameter  $Q_0$  in Eq. 2 ( $b = 1$ ) was determined for each of the three AFX data sets.  $Q_0$  was determined from the fractographic results using a least-square fit of Eq. 2 (Ref. 22). A fractographic crack size range of 0.127 mm - 2.54 mm (0.005 inch - 0.10 inch) was used. An upper bound EIFS of  $X_u = 0.762$  mm (0.03 inch) was assumed for the IFQ distribution. Using Eq. 11 and the estimated  $Q_0$  values, the corresponding lower bound of TCI value,  $\epsilon$ , for each reference crack size,  $a_0$ , was determined for each data set. The results of  $Q_0$  and  $\epsilon$  are shown in Table 2.

Table 2 - Summary of IFQ model parameters for fighter spectrum

DATA SET	$\sigma_{MAX}$ (MPa)	$a_0$ (mm)	$Q_0 \times 10^4$ (HRS <sup>-1</sup> )	$\epsilon$ (HRS)	$\alpha_0$	$\beta_0$ (HRS)	$Q_0 \beta_0$
AFXLR4	220.7	0.76	1.201	0	1.823	15,033	1.805
		1.27		4,253		12,916	1.551
		2.54		10,025		13,421	1.612
AFXMR4	234.5	0.76	2.037	0		8,721	1.777
		1.27		2,508		7,759	1.581
		2.54		5,910		9,093	1.852
AFXHR4	262.1	0.76	4.731	0		5,469	2.587
		1.27		1,079		5,098	2.412
		2.54		2,545		4,798	2.175
POOLED $\alpha_0 = 1.823$ , AVERAGE $Q_0 \beta_0 = 1.928$							

Fractographic results for the three AFX series data sets were combined together to determine the corresponding "pooled"  $\alpha_0$  value using the following procedures. Time-to-crack-initiation (TCI) results for three different reference crack sizes ( $a_0 = 0.762$  mm, 1.27 mm and 2.54 mm) were used for each of the three data sets. The adjusted TCI data, i.e., TCI- $\epsilon$  data for each reference crack size for each data set were normalized using the corresponding average values ( $\bar{X}$ ). Results for the three data sets were pooled together and the (TCI- $\epsilon$ )/ $\bar{X}$  data were ranked in ascending order. Equation 1 was transformed into a least-squares fit form for determining the pooled  $\alpha_0$  value (Ref. 22). The pooled value was found to be 1.823 (Table 2).

After determining  $\alpha_0$  for the pooled data sets, the adjusted TCI's for each reference crack size for each data set were considered separately to determine the corresponding  $\beta_0$  values (Ref. 22). These values are presented in Table 2. Also summarized in Table 2 are the  $Q_0 \beta_0$  values for the nine cases considered. For generic

EIFS, the  $\tau_0$  and  $Q_0 \beta_0$  values should be constants. Average values of  $Q_0 \beta_0 = 1.928$  and  $\tau_0 = 1.823$  are used for the present durability analysis. A plot of  $Q_0$  versus  $\beta_0$  is shown in Fig. 8 for the three data sets considered (9 cases).

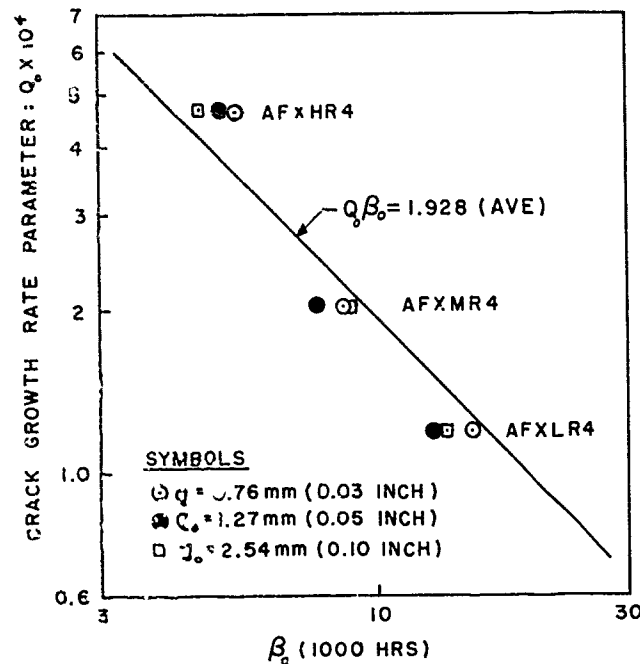


Figure 8 -  $Q_0$  versus  $\beta_0$  for fighter 400-hour load spectrum

The crack growth rate parameter  $Q_i$  for the three data sets is plotted against the applicable gross stress for each data set in Fig. 9. Using a least-square fit (solid line in Figure 9), the following expression is obtained for  $Q_i$  as a function of stress level when stress is expressed in ksi units:

$$Q_i = 1.427 \times 10^{-16} \sigma^{7.928} \quad (20)$$

When stress is expressed in MPa units, the appropriate expression for  $Q_i$  is as follows:

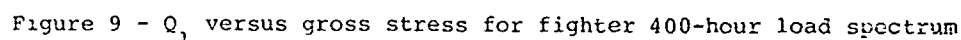
$$Q_i = 3.2 \times 10^{-23} \sigma^{7.928} \quad (21)$$

Equation 20 is used to estimate the  $Q_i$  value for each of the ten stress regions shown in Fig. 6.

Crack exceedance predictions for the fighter lower wing skin were determined using Eqs. 13, 15, and 20 as well as the following parameters:  $x_0 = 0.762 \text{ mm (0.03 inch)}$ ,  $\tau_0 = 1.823$ ,  $Q_0 \beta_0 = 1.928$  (average),  $\ell = 4$  and various  $\tau$  values. The results are presented in various formats as described below.

The extent of damage predictions for the fighter lower wing skin are summarized in Table 3 at  $\tau = 16000$  flight hours for each of the ten stress regions shown in Fig. 6. The number of fastener holes with a crack size  $> 0.762 \text{ mm (0.03 inch)}$ ,  $\bar{L}(\tau)$ , and the standard deviation,  $\sigma_L(\tau)$ , was estimated to be 17.6 and 4.077, respectively. Based on the test results for the right hand and left hand lower wing skins, an average of 16.5 fastener holes had a crack size  $> 0.762 \text{ mm (0.03 inch)}$  at  $\tau = 16000$  hours. In Table 3, the predicted extent of damage results track the average test results for the individual stress regions very well.

In Fig. 10 the predicted percentages of crack exceedance versus fastener hole crack size are plotted for the fighter lower wing skin at  $\tau = 16000$  flight hours. Curves 1, 2 and 3 are based on  $\bar{L}(\tau) \times 100\%/N^*$ ,  $[\bar{L}(\tau) + \sigma_L(\tau)] \times 100\%/N^*$  and  $[\bar{L}(\tau) - \sigma_L(\tau)] \times 100\%/N^*$ , respectively.  $\bar{L}(\tau)$  and  $\sigma_L(\tau)$  are defined by Eqs. 18 and 19, respectively.  $N^*$  is the total number of fastener holes in the fighter lower wing skin (i.e., 1614 holes). Since the number of fastener holes in each stress region is large, it is reasonable to approximate the binomial distribution by the normal distribution. The corresponding exceedance probabilities for curves 1, 2, 3 are shown in Fig. 10 in parentheses.



STRESS REGION	$Q_i \times 10^4$ (HRS <sup>-1</sup> )	$p(i, \tau)$	NO. HOLES WITH $a > 0.76$ mm @ $\tau = 16,000$ HRS				
			PRFDICTED		TEST		
			$\bar{N}(i, \tau)$	$\sigma_N(i, \tau)$	P.H. WING	L.H. WING	AVERAGE
I	0.4620	0.0426	2.5	1.547	7	0	3.5
II	0.3182	0.02182	6.9	2.598	7	2	4.5
III	0.1380	0.00480	3.3	1.812	4	1	2.5
IV	0.0071	0.00002	0.0		0	0	0
V	0.4751	0.0448	0.4	0.618	1	0	0.5
VI	0.5921	0.0662	1.9	1.332	5	1	3.0
VII	1.3504	0.2649	2.1	1.242	0	2	1.0
VIII	0.2507	0.0142	0.1	0.314	1	1	1.0
IX	0.2507	0.0142	0.2	0.444	1	0	0.5
X	0.2152	0.0108	0.2	0.445	0	0	0

$\bar{L}(i) = 17.6, \sigma_L(i) = 4.077, \text{TOTAL TEST AVERAGE} = 16.5$

10-14

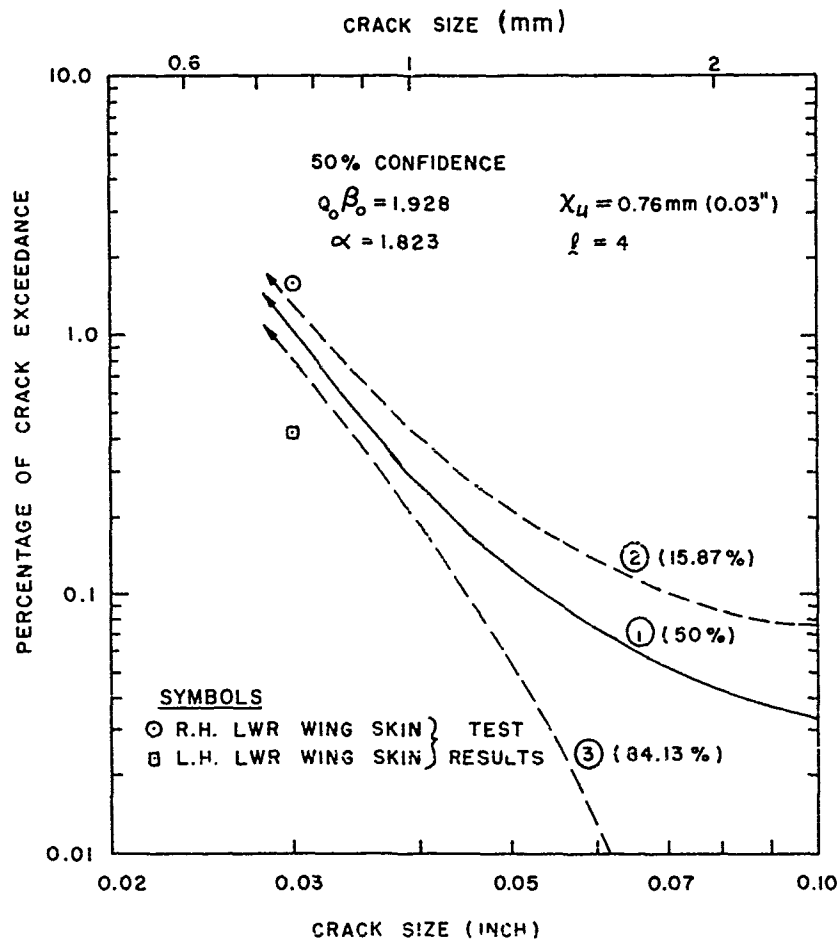


Figure 10 - Percentage of crack exceedance versus crack size at 16,000 hours for 3 probability levels (fighter)

Test results for the right and left hand lower wing skin (at  $X_1 = 0.762 \text{ mm}$  and  $\tau = 16000$  hours) are plotted as a circle and a square, respectively, in Fig. 10. Approximately 1.1% of the fastener holes in the fighter lower wing skin are predicted to have a crack size  $> 0.762 \text{ mm}$  (0.03 inch) at  $\tau = 16000$  hours. This compares with an average of 1.02% based on test results for the right hand and left hand lower wing skins.

In Fig. 10, the predicted average percentage of crack exceedance decreases rapidly for larger crack sizes. For example, the average percentage of crack exceedance for the fighter lower wing skin decreases from approximately 1.1% at  $X_1 = 0.762 \text{ mm}$  (0.03 inch) to approximately 0.14% at  $X_1 = 1.27 \text{ mm}$  (0.05 inch). Crack exceedance predictions are based on the service crack growth master curve defined by Eqs. 13 and 14. A single service crack growth master curve may not adequately fit the full range of desired crack sizes for all crack exceedance predictions. For example, different service crack growth master curves are required to fit two different crack size ranges as illustrated in Fig. 11. Curve 1 and Curve 2 shown in Fig. 11 apply to crack size ranges  $A_1$  and  $A_2$ , respectively. Crack exceedance predictions based on Curves 1 and 2 of Fig. 11 will be different for the same crack exceedance size,  $X_1$ . For example,  $p(i, \tau)$  predictions based on Curve 2 for  $X_1$ ,  $\tau_1$ , and  $X_2$ ,  $\tau_2$  will be larger than those based on Curve 1.

The extrapolation of crack exceedance predictions to larger crack sizes should be consistent with the applicable crack growth process for given design conditions and the crack exceedance crack size,  $X_1$ . Further research is needed to develop a better understanding and confidence in crack exceedance predictions for different crack sizes, materials, and design conditions.

Analytical predictions of the extent of damage are presented in Fig. 12 in an exceedance probability format. In this case, the predicted number of fastener holes in the fighter lower wing skin with a crack size  $\geq 0.762 \text{ mm}$  (0.03 inch) are plotted as a function of flight hours for different exceedance probability values (i.e.,  $P = 0.05, 0.50, 0.95$ ). The plots are based on Eq. 15,  $x_1 = 0.762 \text{ mm}$  (0.03 inch),  $Q_0 B_0 = 1.928$  (average),  $l = 4$ ,  $N_1 = 1614$  fastener holes,  $Z = 11.65$  and  $\bar{L}(\tau) = Z \sigma_L(\tau)$ . For example, at  $\tau = 16,000$  hours,  $\bar{L}(\tau) = 17.6$  fastener holes and  $\sigma_L(\tau) = 4.077$ . The upper bound prediction,  $\bar{L}(\tau) + Z \sigma_L(\tau)$ , is approximately 24.3 fastener holes. In other words, there is a probability of 0.05 that more than 24.3 fastener holes in the fighter lower wing skin will have a crack size  $\geq 0.762 \text{ mm}$  (0.03 inch) at 16000 flight hours. There is a probability of 0.50 and 0.95, respectively, that more than 17.6 and 10.9 fastener holes will have a crack size  $\geq 0.762 \text{ mm}$  (0.03 inch) at  $\tau = 16000$  flight hours. The

average and upper/lower bound predictions for the fighter lower wing skin compare very well with test results for the right hand and left hand lower wing skins at  $\tau = 16000$  flight hours (Fig. 12).

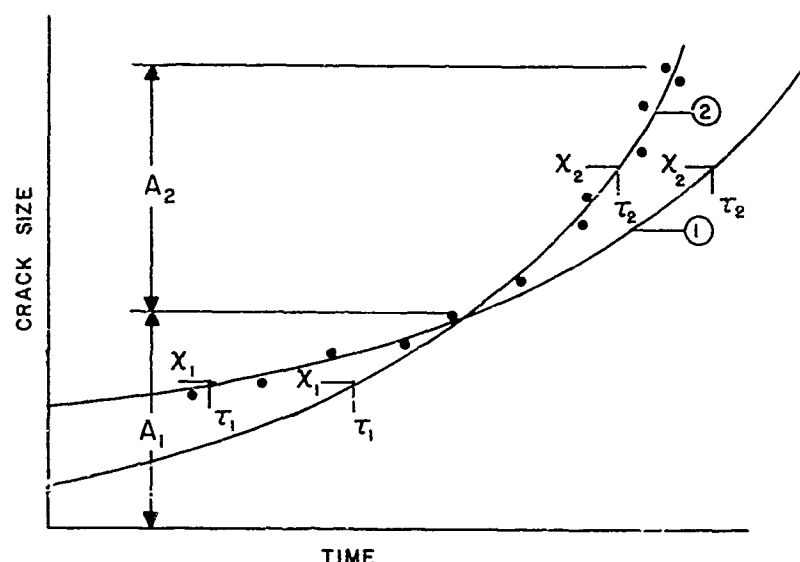


Figure 11 - Service crack growth master curves for different crack size ranges

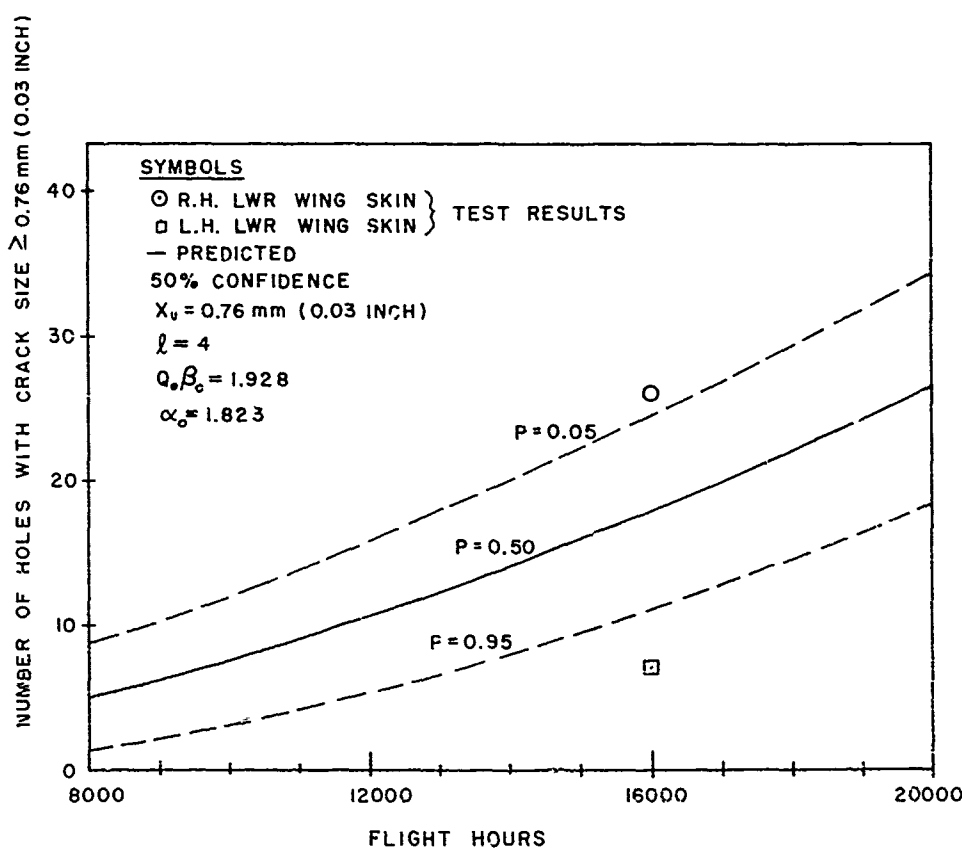


Figure 12 - Number of holes with crack size  $\geq 0.76$  mm (0.03 inch) versus flight hours - exceedance probability format (fighter)

The extent of damage predictions are presented in a stress level format in Fig. 13. Curves are shown for the baseline stress ( $\sigma$ ),  $1.1\sigma$ , and  $1.2\sigma$ . Results are based on Eqs. 13, 15 and 20. The "baseline stress" refers to the maximum stress level for each of the ten stress zones. For prediction purposes, the baseline stresses for each



10-14 stress zone were all increased by the same percentage. The results shown in Fig. 13 can be used to assess the extent of damage as a function of stress level and flight hours. This format is particularly useful for evaluating durability design tradeoffs in terms of the extent of damage. For example, at  $\tau = 16000$  flight hours approximately 1.1% of the fastener holes in the fighter lower wing skin would be predicted to exceed a crack size of 0.762 mm (0.03 inch) for the baseline stress levels. If the baseline stresses were increased to  $1.1\sigma$  and  $1.2\sigma$ , the predicted average percentage of holes with a crack size  $\geq 0.762$  mm (0.03 inch) would be approximately 4% and 12%, respectively. This provides a quantitative measure of the structural durability as a function of stress level and flight hours.

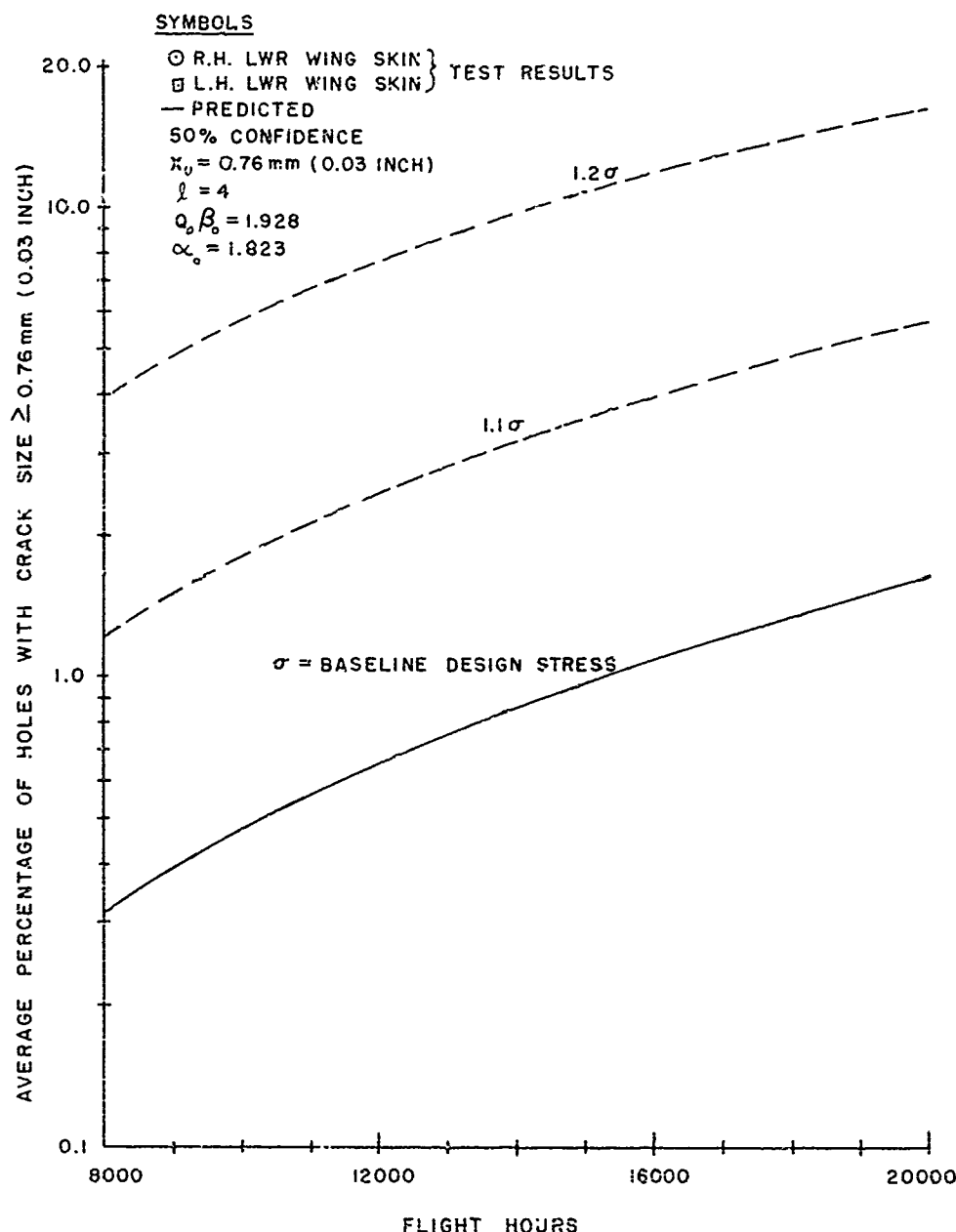


Figure 13 - Average percentage of holes with crack size  $\geq 0.76$  mm (0.03 inch) versus flight hours - stress level format (fighter)

#### B. Complex-splice Specimens Subjected to Bomber Load Spectrum

A durability analysis of complex-splice specimens subjected to a bomber load spectrum is presented. Analytical predictions of the extent of damage in the specimens are presented in various formats and compared with fractographic results. The analytical/experimental results are summarized here and described in more detail in Ref. 27.

The complex-splice specimen geometry is presented in Fig. 14. The specimens were made of 7475-T7351 aluminum plate and countersunk steel rivets. A bomber load spectrum was applied. Based on a simplified stress analysis and strain gage results, the maximum gross stress in the outer row of fastener holes at the faying surface was

estimated to be 246.8 MPa (35.8 ksi). The eleven specimens were tested to two service lifetimes (27,000 flight hours) or failure, whichever came first.

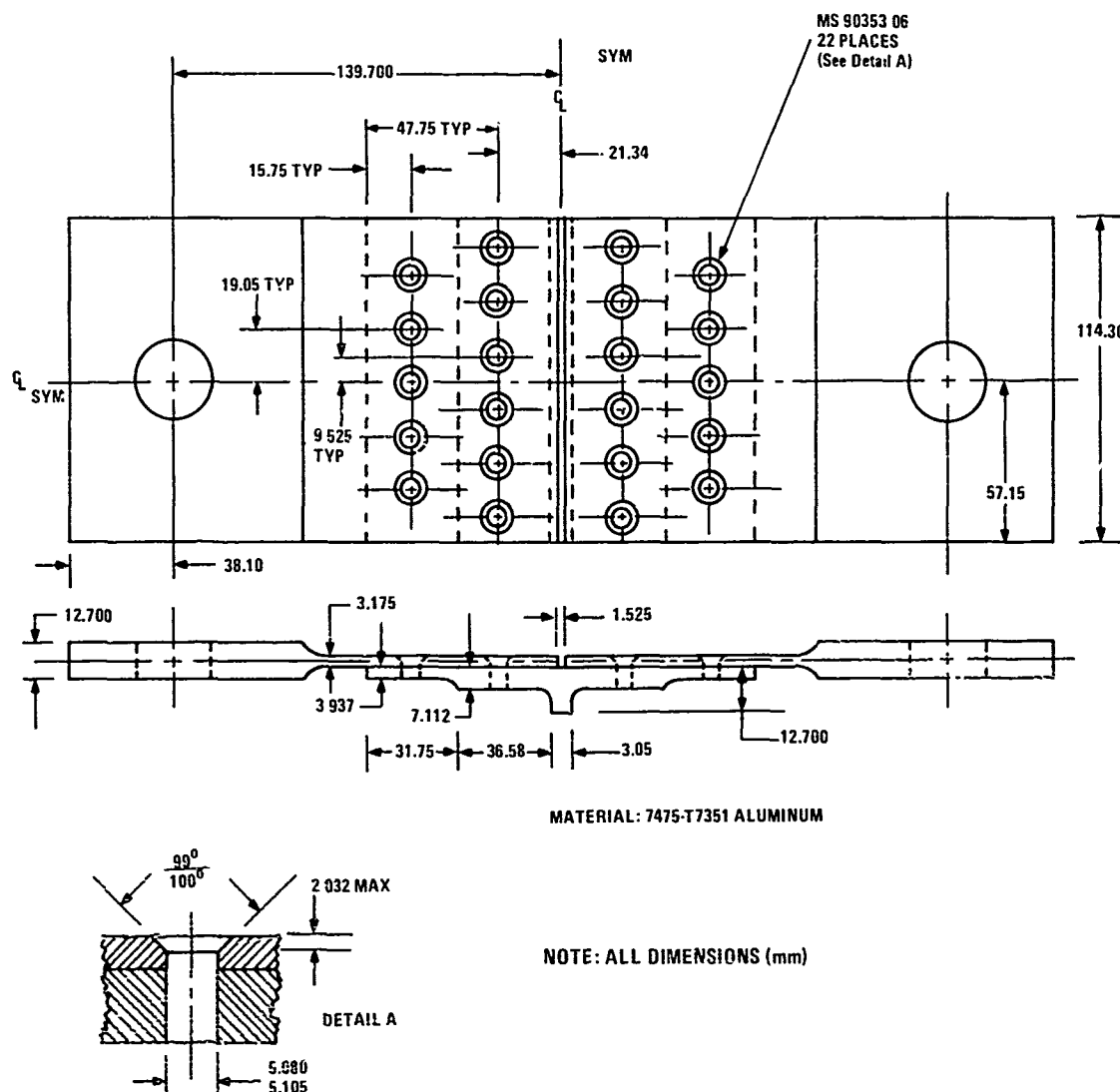


Figure 14 - Complex - splice specimen

After testing, all fastener holes in the outer rows were inspected. Fractography was performed for the largest crack in each fastener hole in the outer rows. Twenty five out of 110 fastener holes in the outer rows had a crack size  $\geq 1.27$  mm (.05 inch) at 13,500 hours. Hence, 22.7% of the fastener holes in the outer rows had a crack size  $\geq 1.27$  mm (.05 inch) at 13,500 hours.

The IFQ of the fastener holes was based on the fractographic results for nine data sets and three different reference crack sizes. The specimens were made of 7475-T7351 aluminum and contained 2 countersunk rivets. Load transfer levels of 15%, 30% and 40% were considered. All specimens had the same configuration (fig. 7) with the same overall length and basic test section dimensions. However, the lug end dimensions varied depending on the amount of load transfer. Three maximum stress levels were considered for each load transfer level.

A fractographic crack size range of 0.127 mm - 2.54 mm (0.005 inch - 0.1 inch) was considered. An upper bound SIFS of  $X_0 = 1.27$  mm (0.05 inch) was assumed for the IFQ distribution. A fractography scaling factor of  $\lambda = 4$  was used. The same data pooling procedures were used which were previously described for the lighter demonstration. The average  $\alpha_0$  and  $Q_0 R_0$  values were found to be 2.702 and 2.823, respectively.

10-18

The crack growth rate parameter  $Q_i$  for the 9 data sets is plotted against the applicable gross stress for each data set in Fig. 15. The solid line represents the least-square best fit through the plot points. The dashed lines have the same slope as the solid line and they encompass all the plot points. The corresponding best-fit equation for  $Q_i$  as a function of gross stress level when stress is expressed in ksi units is as follows:

$$Q_i = 6.151 \times 10^{-13} \sigma^{5.381} \quad (22)$$

When stress is expressed in MPa units, the appropriate expression for  $Q_i$  is as follows:

$$Q_i = 1.895 \times 10^{-17} \sigma^{5.381} \quad (23)$$

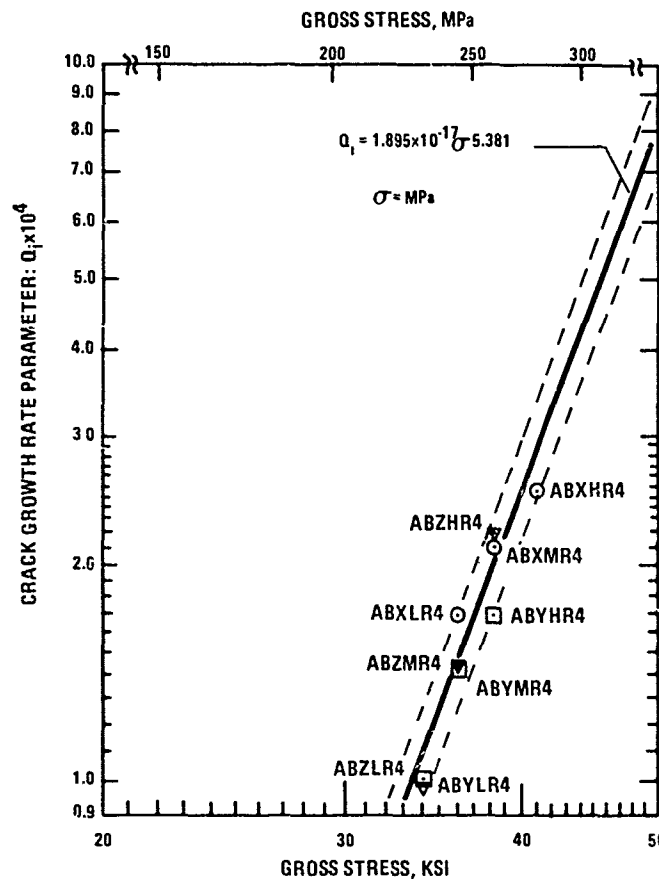


Figure 15 -  $Q_i$  versus gross stress for bomber load spectrum

Crack exceedance predictions for the complex-splice specimens were determined using Eqs. 13, 15, and 22. At  $\tau = 13,500$  hours, an average of 9 fastener holes (8.3%) were predicted to exceed a crack size of 1.27 mm (0.05 inch). The test results showed an average of 25 fastener holes (22.7%) exceeding a crack size of 1.27 mm (0.05 inch). The difference in the predicted and test crack exceedances is attributed mainly to the stress level used in the predictions. The actual stress level and distribution in the outer row of fastener holes is far more complex, due to lateral bending effects, than those considered for the damage assessment. The crack exceedance predictions are very sensitive to the gross applied stress level used. This is illustrated in Fig. 16. The solid line represents average crack exceedance predictions for the gross stress level of 246.8 MPa (35.8 ksi) obtained using the simplified stress analysis approach. The dashed lines represent average crack exceedance predictions for other gross stress levels. Also plotted as a single point is the average test crack exceedance at  $\tau = 13,500$  hours. It can be seen that if the gross applied stress level used in the predictions were 266.1 MPa (38.6 ksi) rather than 246.8 MPa (35.8 ksi), the predicted crack exceedance at  $\tau = 13,500$  hours would match the test results. Hence, a more accurate stress analysis could result in improved predictions.

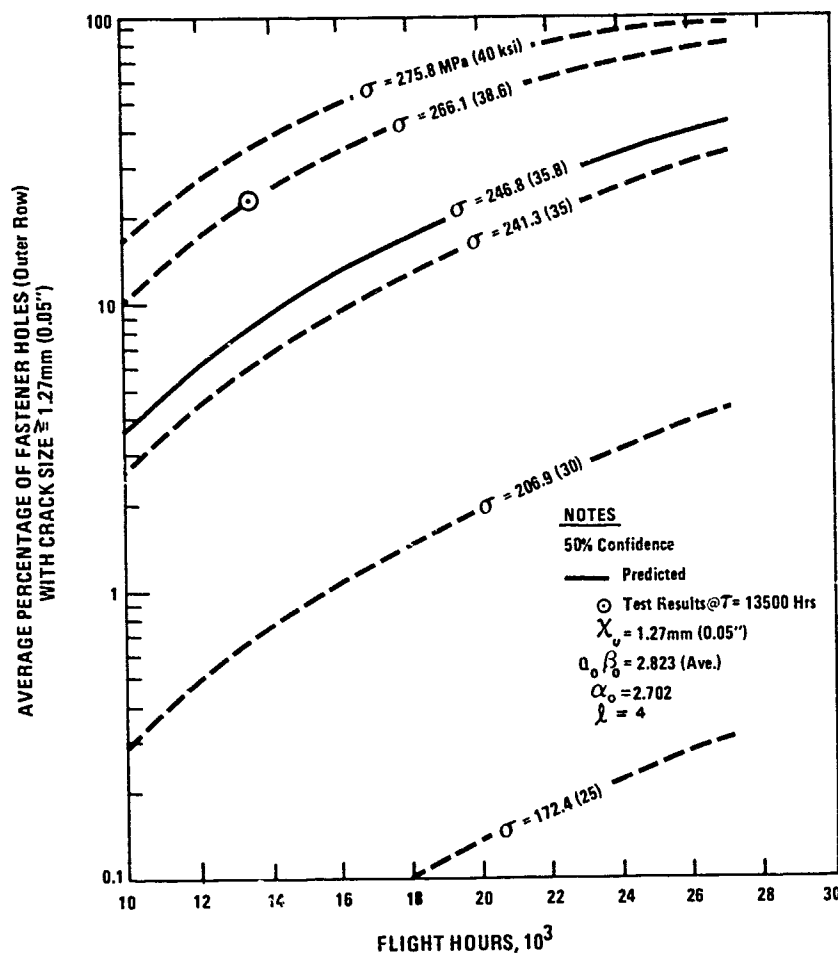


Figure 16 - Average percentage holes with crack size  $> 1.27 \text{ mm}$  (0.05 inch) versus flight hours stress level format (bomber)

Other useful crack exceedance formats, previously discussed for the fighter demonstration, are presented in Figs. 17 and 18 for the complex-splice specimens.

## VI CONCLUSIONS AND DISCUSSIONS

Probabilistic fracture mechanics methods for durability analysis have been described and demonstrated for both a full-scale fighter aircraft structure and for a complex splice subjected to a bomber spectrum. These methods can be used to analytically assure compliance with the Air Force's durability design requirements. The analytical tools described can be used to quantify the extent of damage as a function of the durability design variables for structural details in a part, a component or airframe. Once the economic life and durability critical parts criteria are established, the extent of damage predictions can be used to assure design compliance with Air Force durability requirements.

An initial fatigue quality model can be used to define the EIFS cumulative distribution using suitable fractographic results. Procedures and guidelines have been developed for determining the IFQ model parameters for pooled fractographic data sets and for scaling TSCI results. The parameters  $\alpha_0$  and  $Q_0 \beta_0$  provide the basis for putting fractographic results on a common baseline for quantifying the initial fatigue quality. For generic EIFS,  $\alpha_0$  and  $Q_0 \beta_0$  should be constants for different fractographic data sets (same material, fastener type/fit, and drilling technique), loading spectra, stress levels and percent load transfer. Encouraging results have been obtained to justify the use of the same EIFS cumulative distribution for crack exceedance predictions for different design conditions. Further research is required to confirm the IFQ distributions for different materials, load spectra, stress levels, fastener types/diameters/fit, % load transfer, etc. A considerable amount of fractographic results exist which need to be evaluated using the IFQ model.

The effects of fretting, clamp-up, corrosion, size effect (scale-up from coupon to component), faying surface sealant, interference-fit fasteners, etc. on IFQ need to be investigated. Also, the feasibility of using no-load transfer specimens with multiple holes for quantifying the IFQ should be evaluated using spectrum and constant amplitude loading. This could provide an economical way to generate the fractographic results needed to quantify the IFQ.

10-20

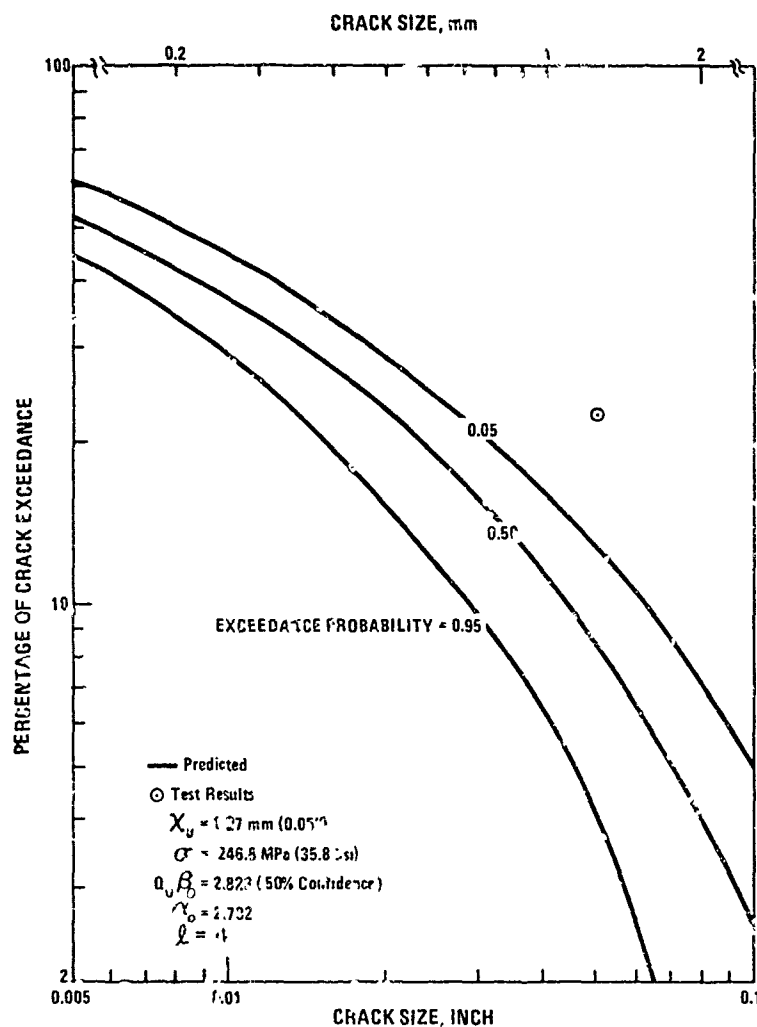


Figure 17 - Percentage of crack exceedance versus crack size and exceedance probability at 13,500 hours (bomber)

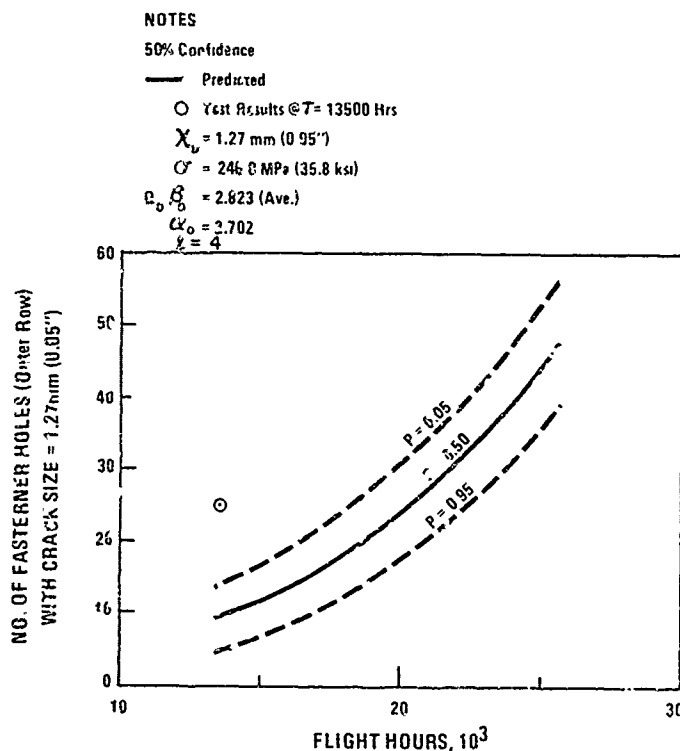


Figure 18 - Number of holes with crack size > 1.27 mm (0.05 inch) versus flight hours - exceedance probability format (bomber)

Theoretically, the IFQ model can be used to quantify the EIFS cumulative distribution for various structural details as long as fractographic results are available for the details to be included in the durability analysis. The IFQ model has been evaluated using fractographic results for fastener holes. Suitable specimens and guidelines need to be developed for generating crack initiation and crack growth results for other details such as, cutouts, fillets, lugs, etc. Fractographic results should be developed and evaluated for such details so that the durability analysis methods described can be efficiently applied to different types of structural details in typical aircraft structures.

The accuracy of crack exceedance predictions, based on the same EIFS cumulative distribution, needs to be evaluated for different design conditions. Also, IFQ model parameter sensitivity studies need to be performed to better understand the average parameter values and variances and the impact of these parameters on the IFQ for different fractographic data sets.

The durability analysis methodology was developed for crack exceedance predictions for relatively small crack sizes (e.g., = 2.54 mm) in structural details. The largest crack in each detail was assumed to be statistically independent to justify using the binomial distribution for combining crack exceedance predictions for structural details. If the largest crack in a given detail doesn't significantly affect the growth of cracks in neighboring details, perhaps the proposed durability analysis methodology can be extended to crack sizes >2.54 mm (0.10 inch). The simplistic crack growth rate equation (Eq. 12) is not suitable for use in the crack exceedance predictions for crack sizes >2.54 mm (0.10 inch). However, a general service crack growth master curve can be generated under given design conditions which is valid for crack sizes >2.54 mm (Ref. 11, 12 15, and 22). Nevertheless, this approach has not been demonstrated in the present study and further research is required to extend the probabilistic fracture mechanics approach developed to larger crack sizes.

Two different  $F_a(0)(x)$  equations (i.e., Eqs. 7 and 9) were presented for representing the IFQ. Either equation works but Eq. 9 is recommended for two reasons: (1) It assures all EIFS's in the IFQ distribution will be >0, and (2) the crack growth rate parameter  $Q_0$  can be easily determined from the fractographic results and the resulting  $Q_0$  values for different data sets will be directly comparable. If Eq. 7 is used, a common b parameter (Eq. 2) must be imposed for different fractographic data sets to put the  $Q_0$  values on a comparable baseline. As long as  $b > 1$ , all EIFS's in Eq. 7 will be  $\geq 0$ . Further studies are needed to evaluate the accuracy of these two  $F_a(0)(x)$  equations.

The EIFS cumulative distribution,  $F_a(0)(x)$ , is independent of the reference crack size,  $a_0$ . This is illustrated in Eqs. 7 and 9. Therefore, the TPCI distribution for different reference crack sizes will transform into a common  $F_a(0)(x)$ .

The IFQ model is simply a "mathematical tool" for quantifying the IFQ of structural details. Therefore, the resulting EIFS's must be considered in the context of the IFQ model and the fractographic results used to calibrate the model parameters. EIFS's should be considered as hypothetical cracks used for crack exceedance predictions rather than actual initial flaws per se.

Back extrapolations of fractographic data must be done consistently to put the EIFS's on a common baseline for different data sets. Inconsistent EIFS results will be obtained if the EIFS distribution is determined by back extrapolating the fractography results for individual specimens and then fitting a statistical distribution to the EIFS results for different data sets. Two problems result in this approach is used: (1) the EIFS's are not on a common baseline for different data sets, and (2) the resulting EIFS distribution is not statistically compatible with the TPCI distribution and the fatigue wear out process. The resulting EIFS distribution should be statistically compatible with the TPCI distribution. The IFQ model presented in this paper satisfies this requirement.

Several useful applications of the durability analysis methodology developed are: (1) The evaluation of durability design tradeoffs in terms of structural design variables, (2) the evaluation of structural maintenance requirements before or after aircraft is committed to service, and (3) the evaluation of aircraft user options affecting life-cycle costs, structural maintenance requirements, and operational readiness.

#### ACKNOWLEDGEMENTS

This research is based on work performed under the "Durability Methods Development" program, sponsored by the Air Force Wright Aeronautical Laboratories, Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio (Contract F33615-77-C-3123). The authors are indebted to the following General Dynamics' personnel who supported the research effort: D.D. Gordon, J.W. Norris, S.M. Speaker, W.T. Kaarlela, A. Meder, R.O. Nay and F.C. Nord. Valuable discussions with Dr. Y.H. Kim, Dr. W.R. Garver, Dr. V.D. Smith and J.W. Morrow of General Dynamics and Dr. M. Shinozuka of Modern Analysis Incorporated are acknowledged.

## REFERENCES

1. Anon., Aeronautical Systems Division, "Aircraft Structural Integrity Program," 1975, MIL STD 1530A.
2. Anon., Aeronautical Systems Division, "Airplane Damage Tolerance Requirements," 1974, MIL-A-83444 (USAF).
3. Anon., Aeronautical Systems Division, "Airplane Strength and Rigidity Ground Tests," 1975, MIL-A-8867B.
4. Anon., Aeronautical Systems Division, "Airplane Strength, Rigidity and Reliability Requirements; Repeated Loads and Fatigue," 1975, MIL-A-8866B.
5. Pendley, B.J., Henslee, S.P., and Manning, S.D., Air Force Flight Dynamics Laboratory, "Durability Methods Development, Volume III - Structural Durability Survey: State-Of-The-Art Assessment," 1979, AFFDL-TR-79-3118.
6. Wood, H.A., Engle, R.M., Gallagher, J.P., and Potter, J.M., Air Force Flight Dynamics Laboratory, "Current Practice on Estimating Crack Growth Damage Accumulation with Specific Application to Structural Safety, Durability and Reliability," 1976, AFFDL-TR-75-32.
7. Tiffany, C.F. et al, American Institute for Aeronautics and Astronautics, "Analysis of USAF Aircraft Structural Durability and Damage Tolerance," 1978, Proceedings of Structural Durability and Damage Tolerance Workshop.
8. Corfin, M.D., and Tiffany, C.F., "New Air Force Requirements for Structural Safety, Durability, and Life Management," Journal of Aircraft, Vol. 13, No. 2, 1976, pp. 93-98.
9. Yang, J.N., "Statistical Estimation of Service Cracks and Maintenance Cost for Aircraft Structures," Journal of Aircraft, Vol. 13, No. 12, 1976, pp. 929-937.
10. Yang, J.N., "Statistical Approach to Fatigue and Fracture Including Maintenance Procedures," Fracture Mechanics, Univ. of Virginia Press, Proc. of 2nd International Conf. on Fracture Mechanics, 1973, pp. 559-577.
11. Yang, J.N., "Statistical Estimation of Economic Life for Aircraft Structures," Journal of Aircraft, Vol. 17, No. 7, 1980, pp. 528-535.
12. Manning, S.D., Yang, J.N., Shinozuka, M., and Garver, W.R., et al, Air Force Flight Dynamics Laboratory, "Durability Methods Development, Volume I - Phase I Summary," 1979, AFFDL-TR-79-3118.
13. Manning, S.D., Flanders, M.A., Garver, W.R., and Kim, Y.H., Air Force Flight Dynamics Laboratory "Durability Methods Development, Volume II - Durability Analysis: State-Of-The-Art- Assessment," 1979, AFFDL-TR-79-3118.
14. Shinozuka, M., Air Force Flight Dynamics Laboratory, "Durability Methods Development, Volume IV - Initial Quality Representation," 1979, AFFDL-TR-79-3118.
15. Yang, J.N., Manning, S.D., and Garver, W.R., Air Force Flight Dynamics Laboratory, "Durability Methods Development, Volume V - Durability Analysis Methodology Development," 1979, AFFDL-TR-79-3118.
16. Yang, J.N., and Manning, S.D., "Statistical Distribution of Equivalent Initial Flaw Size," 1980 Proceedings Annual Reliability and Maintainability Symposium, 1980, pp. 112-120.
17. Walker, E.K., Ekvall, J.C., and Rhodes, J.E., "Design for Continuing Structural Integrity," Trans. ASME, Vol. 102, January 1980, pp. 32-39.
18. Manning, S.D., and Smith, V.D., "Economic Life Criteria for Metallic Airframes," Proceedings of 21st AIAA Structures, Structural Dynamics, and Materials Conference, Part 1, 1980, pp. 504-511.
19. Yang, J.N., "Statistical Crack Growth in Durability and Damage Tolerant Analyses," Proceedings of the AIAA/ASME/ASCE/AHS 22nd Structures, Structural Dynamics and Materials Conference, Part 1, 1981, pp. 38-49.
20. Manning, S.D., Garver, W.R., Kim, Y.H., and Rudd, J.L., "Durability Analysis - Format Requirements and Critique of Potential Approaches," 1981 Proceedings of ASME Failure Prevention and Reliability Conference, 1981, pp. 223-229.

21. Rudd, J.L., Yang, J.N., Manning, S.D., and Garver, W.R., "Durability Design Requirements and Analysis for Metallic Airframes," Design of Fatigue and Fracture Resistant Structures, ASTM STP 761, 1982, pp. 133-151.
22. Manning, S.D., Yang, J.N., et al, Air Force Flight Dynamics Laboratory, "Durability Methods Development, Volume VII - Phase II Documentation," 1982, AFFDL-TR-79-3118.
23. Speaker, S.M., and Gordon, D.E., Air Force Flight Dynamics Laboratory, "Durability Methods Development, Volume VIII - Test and Fractography Data," 1982, AFFDL-TR-79-3118.
24. Norris, J.W., Air Force Flight Dynamics Laboratory, "Durability Methods Development, Volume IX - Documentation of Durability Analysis Computer Program," to be published, AFFDL-TR-79-3118.
25. Manning, S.D., and Yang, J.N., Air Force Flight Dynamics Laboratory, "Durability Methods Development, Volume X - Final Program Summary," to be published, AFFDL-79-3118.
26. Manning, S.D., Norris, J.W., and Yang, J.N., Air Force Flight Dynamics Laboratory, "USAF Durability Design Handbook: Guidelines for the Analysis and Design of Durable Aircraft Structures," to be published.
27. Rudd, J.L., Yang, J.N., Manning, S.D., and Yee, B.G.W., "Damage Assessment of Mechanically Fastened Joints in the Small Crack Size Range," Proceedings of the Ninth U.S. National Congress of Applied Mechanics, 1982.
28. Freudenthal, A.M., "The Expected Time To First Failure," Air Force Materials Laboratory, AFML-TR-66-37, February 1966.
29. Freudenthal, A.M., Itagahi, H., and Shinozuka, M., "Time to First Failure for Various Distributions of Time To Failure," Air Force Materials Laboratory, AFML-TR-66-241, July 1966.
30. Yang, J.N., and Trapp, W.J., "Joint Aircraft Loading/Structure Response Statistics of Time to Service Crack Initiation," Journal of Aircraft, AIAA, Vol. 13, No. 4, 1976, pp. 270-278.
31. Rudd, J.L., and Gray, T.D., "Equivalent Initial Quality Method," Air Force Flight Dynamics Lab., AFFDL-TM-76-83, 1976.
32. Rudd, J.L., and Gray, T.D. "Quantification of Fastener Hole Quality," Proc. 18th AIAA/ASME/SAE Structures, Structural Dynamics and Materials Conf., 1977.
33. Norcna, P.J., et al, "Fastener Hole Quality," Vol. I & II, Air Force Flight Dynamics Lab., AFFDL-TR-78-209, 1978.
34. Potter, J.M., "Advances in Fastener Hole Quality Through the Application of Solid Mechanics," in Proceedings of U.S. Army Symposium on Case Studies in Structural Integrity and Reliability, AMMRC-MS-78-3, U.S. Army Mechanics and Material Research, Watertown, MA., 1978.
35. Wood, H.A., Rudd, J.L., and Potter, J.M., "Evaluation of Small Cracks in Aerospace Structures," AGARD Publication (In Press) CESML, Turkey, 1981.
36. Johnson, W.S., and Spamer, T., "A User's Guide to CGR-GD, A Computerized Crack Growth Prediction Program," General Dynamics, Fort Worth Division, Report FZS-241, November 1976.
37. "Test Results of Spectrum/Environmental Fatigue Testing to Determine the Crack Growth Rates of Flaws in 7475, 2124, and 2024 Aluminum Alloys and 6Al-4V-Titanium," General Dynamics, Fort Worth Division, Report 16PR925, 10 June 1978.